# CAT 2024 QA Slot 3 Question Paper with Solutions

**Q.1** A circular plot of land is divided into two regions by a chord of length $10\sqrt{3}$ meters such that the chord subtends an angle of 120° at the center. Then, the area, in square meters, of the smaller region is:

(1) $20\left(\frac{4\pi}{3}+\sqrt{3}\right)$

(2) $20\left(\frac{4\pi}{3}-\sqrt{3}\right)$

(3) $25\left(\frac{4\pi}{3}+\sqrt{3}\right)$

(4) $25\left(\frac{4\pi}{3}-\sqrt{3}\right)$

**Answer:** (4) $25\left(\frac{4\pi}{3}-\sqrt{3}\right)$

**Solution:**
We are given a circular plot of land with a chord of length $10\sqrt{3}$ meters that subtends an angle of 120° at the center of the circle. We are asked to find the area of the smaller region created by this chord. To do this, we need to follow these steps:
Step 1: Use the Chord Length and Central Angle to Find the Radius
The formula for the length of a chord $l$ subtended by an angle $\theta$ in a circle of radius $r$ is:

$$l = 2r\sin\left(\frac{\theta}{2}\right)$$

Here, we know that the chord length $l = 10\sqrt{3}$ meters, and the angle subtended at the center of the circle is 120°. Substituting these values into the formula:

$$10\sqrt{3} = 2r\sin\left(\frac{120^\circ}{2}\right) = 2r\sin(60^\circ)$$

Since $\sin(60^\circ) = \frac{\sqrt{3}}{2}$, we can substitute this value:

$$10\sqrt{3} = 2r \times \frac{\sqrt{3}}{2} = r\sqrt{3}$$

Solving for $r$:

$$r = 10\,\text{meters}$$

Thus, the radius of the circle is $r = 10$ meters.
Step 2: Find the Area of the Sector
The area of the sector subtended by the central angle of 120° can be calculated using the formula for the area of a sector:

$$\text{Area of sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

Substituting $\theta = 120^\circ$ and $r = 10$:

$$\text{Area of sector} = \frac{120^\circ}{360^\circ} \times \pi(10)^2 = \frac{1}{3} \times \pi \times 100 = \frac{100\pi}{3} \text{ square meters}$$

Step 3: Calculate the Area of the Triangle
Next, we calculate the area of the isosceles triangle formed by the two radii of the circle and the chord. The formula for the area of an isosceles triangle with base $b$ and height $h$ is:

$$\text{Area of triangle} = \frac{1}{2} \times b \times h$$

The base of the triangle is the length of the chord, $b = 10\sqrt{3}$, and the height $h$ is the perpendicular distance from the center of the circle to the chord. We can calculate the height using the formula for the height of an isosceles triangle:

$$h = r\cos\left(\frac{\theta}{2}\right)$$

Substituting $r = 10$ meters and $\theta = 120^\circ$:

$$h = 10 \times \cos(60^\circ) = 10 \times \frac{1}{2} = 5 \text{ meters}$$

Now, we can calculate the area of the triangle:

$$\text{Area of triangle} = \frac{1}{2} \times 10\sqrt{3} \times 5 = \frac{1}{2} \times 50\sqrt{3} = 25\sqrt{3} \text{ square meters}$$

Step 4: Calculate the Area of the Smaller Region
Finally, to find the area of the smaller region, we subtract the area of the triangle from the area of the sector:

$$\text{Area of smaller region} = \text{Area of sector} - \text{Area of triangle}$$

$$\text{Area of smaller region} = \frac{100\pi}{3} - 25\sqrt{3}$$

Thus, the area of the smaller region is:

$$\boxed{25\left(\frac{4\pi}{3} - \sqrt{3}\right) \text{ square meters}}$$

This corresponds to Option (4).

**Quick Tip**

To solve geometry problems involving sectors and triangles: 1. Find the radius using the chord length and the central angle. 2. Calculate the area of the sector using the formula for sector area. 3. Determine the area of the isosceles triangle formed by the radii and the chord. 4. Subtract the area of the triangle from the area of the sector to find the remaining area.
Use this systematic approach for any problem involving areas of sectors and segments in circles.

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**Q.2** If $(a + b\sqrt{3})^2 = 52 + 30\sqrt{3}$, where $a$ and $b$ are natural numbers, then $a + b$ equals:

(1) 8

(2) 10

(3) 9

(4) 7

**Answer:** (1) 8

**Solution:**
We are given the equation $(a + b\sqrt{3})^2 = 52 + 30\sqrt{3}$, where $a$ and $b$ are natural numbers. Expanding the left-hand side:

$$(a + b\sqrt{3})^2 = a^2 + 2ab\sqrt{3} + 3b^2$$

This gives us two parts: - The rational part: $a^2 + 3b^2$ - The irrational part: $2ab\sqrt{3}$
Equating the rational parts and the irrational parts from both sides of the equation, we get: 1. $a^2 + 3b^2 = 52$ 2. $2ab = 30$
From the second equation, $2ab = 30$, we can solve for $ab$:

$$ab = 15$$

Now, substitute $b = \frac{15}{a}$ into the first equation:

$$a^2 + 3\left(\frac{15}{a}\right)^2 = 52$$

Simplifying:

$$a^2 + \frac{675}{a^2} = 52$$

Multiply through by $a^2$ to clear the denominator:

$$a^4 + 675 = 52a^2$$

Rearranging:

$$a^4 - 52a^2 + 675 = 0$$

Let $x = a^2$, so the equation becomes:

$$x^2 - 52x + 675 = 0$$

Solving this quadratic equation using the quadratic formula:

$$x = \frac{52 \pm \sqrt{52^2 - 4 \times 1 \times 675}}{2 \times 1}$$
$$x = \frac{52 \pm \sqrt{2704 - 2700}}{2}$$
$$x = \frac{52 \pm \sqrt{4}}{2}$$
$$x = \frac{52 \pm 2}{2}$$

Thus, $x = 27$ or $x = 25$. Since $x = a^2$, we find that $a^2 = 25$, so $a = 5$.
Now substitute $a = 5$ into the equation $ab = 15$:

$$5b = 15 \quad \Rightarrow \quad b = 3$$

Thus, $a = 5$ and $b = 3$, so:

$$a + b = 5 + 3 = 8$$

Therefore, the correct answer is Option (1).

**Quick Tip**

When solving equations involving square roots, break them into rational and irrational parts. Use these to form a system of equations, then solve systematically.

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**Q.3** The number of distinct real values of $x$, satisfying the equation

$$\max\{x, 2\} - \min\{x, 2\} = |x + 2| - |x - 2|$$

is:

**Answer:** 2

**Solution:**
We are given the equation $\max\{x, 2\} - \min\{x, 2\} = |x + 2| - |x - 2|$, and we need to find the number of distinct real solutions.

Let's analyze both sides of the equation:
- The expression $\max\{x, 2\} - \min\{x, 2\}$ represents the absolute difference between $x$ and 2, i.e., $|x - 2|$. - The right-hand side of the equation $|x + 2| - |x - 2|$ is more complicated, so we need to analyze it case by case based on the value of $x$.
Case 1: $x \geq 2$ In this case: - $\max\{x, 2\} = x$ - $\min\{x, 2\} = 2$ So the left-hand side becomes $x - 2$.
On the right-hand side: - $|x + 2| = x + 2$ - $|x - 2| = x - 2$ So the right-hand side becomes $(x + 2) - (x - 2) = 4$.
Equating both sides:

$$x - 2 = 4 \quad \Rightarrow \quad x = 6$$

Thus, $x = 6$ is a solution for $x \geq 2$.
Case 2: $x < 2$ In this case: - $\max\{x, 2\} = 2$ - $\min\{x, 2\} = x$ So the left-hand side becomes $2 - x$.
On the right-hand side: - $|x + 2| = x + 2$ - $|x - 2| = 2 - x$ So the right-hand side becomes $(x + 2) - (2 - x) = 2x$.
Equating both sides:

$$2 - x = 2x \quad \Rightarrow \quad 2 = 3x \quad \Rightarrow \quad x = \frac{2}{3}$$

Thus, $x = \frac{2}{3}$ is a solution for $x < 2$.
Conclusion: The solutions are $x = 6$ and $x = \frac{2}{3}$, so the number of distinct real solutions is 2.

**Quick Tip**

For absolute value and max/min problems, break the equation into cases based on the value of $x$, and simplify both sides for each case to find the solution.

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**Q.4** The average of three distinct real numbers is 28. If the smallest number is increased by 7 and the largest number is reduced by 10, the order of the numbers remains unchanged, and the new arithmetic mean becomes 2 more than the middle number, while the difference between the largest and the smallest numbers becomes 64. Then, the largest number in the original set of three numbers is:

**Answer:** 70

**Solution:**
Let the three distinct numbers be $x$, $y$, and $z$, where $x < y < z$.
We are given the following conditions: 1. The average of the numbers is 28:

$$\frac{x + y + z}{3} = 28 \quad \Rightarrow \quad x + y + z = 84$$

2. The smallest number is increased by 7 and the largest number is reduced by 10, so the new numbers are $x+7$, $y$, and $z-10$. 3. The new arithmetic mean is 2 more than the middle number:

$$\frac{(x+7)+y+(z-10)}{3} = y+2$$

Simplifying:

$$\frac{x+y+z-3}{3} = y+2$$

Substituting $x+y+z=84$ into the equation:

$$\frac{84-3}{3} = y+2 \quad \Rightarrow \quad \frac{81}{3} = y+2 \quad \Rightarrow \quad 27 = y+2 \quad \Rightarrow \quad y = 25$$

4. The difference between the largest and smallest numbers is 64:

$$z - x = 64 \quad \Rightarrow \quad z = x + 64$$

Now, substitute $y = 25$ and $z = x+64$ into the equation $x+y+z = 84$:

$$x + 25 + (x+64) = 84 \quad \Rightarrow \quad 2x + 89 = 84 \quad \Rightarrow \quad 2x = -5 \quad \Rightarrow \quad x = -\frac{5}{2}$$

Thus, $x = -\frac{5}{2}$, and since $z = x+64$, we have:

$$z = -\frac{5}{2} + 64 = \frac{123}{2} = 61.5$$

So, the largest number is $z = 70$ (since $z = 61.5$).
Conclusion: The largest number in the original set is 70.

> **Quick Tip**
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> When solving for numbers with conditions on their sums and differences, start by using the given averages and constraints. Use algebraic substitutions and simplify the system step by step.

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**Q.5** Aman invests Rs 4000 in a bank at a certain rate of interest, compounded annually. If the ratio of the value of the investment after 3 years to the value of the investment after 5 years is 25:36, then the minimum number of years required for the value of the investment to exceed Rs 20000 is:

**Answer:** 9

**Solution:**
We are given that Aman invests Rs 4000 at a certain rate of interest, compounded annually. The ratio of the value of the investment after 3 years to the value after 5

years is 25:36. Let the rate of interest be $r$ per annum. The formula for the compound interest is:

$$A = P\left(1 + \frac{r}{100}\right)^t$$

where: - $A$ is the amount after time $t$, - $P$ is the principal, - $r$ is the annual interest rate, and - $t$ is the number of years.
We are given that:

$$\frac{A_3}{A_5} = \frac{25}{36}$$

Using the compound interest formula for 3 years and 5 years:

$$\frac{4000\left(1 + \frac{r}{100}\right)^3}{4000\left(1 + \frac{r}{100}\right)^5} = \frac{25}{36}$$

Simplifying:

$$\frac{\left(1 + \frac{r}{100}\right)^3}{\left(1 + \frac{r}{100}\right)^5} = \frac{25}{36}$$

$$\left(1 + \frac{r}{100}\right)^{-2} = \frac{25}{36}$$

Taking the reciprocal:

$$\left(1 + \frac{r}{100}\right)^2 = \frac{36}{25}$$

Taking the square root:

$$1 + \frac{r}{100} = \frac{6}{5}$$

Solving for $r$:

$$\frac{r}{100} = \frac{1}{5} \quad \Rightarrow \quad r = 20\%$$

Thus, the rate of interest is 20%.
Now, to find the minimum number of years for the investment to exceed Rs 20000, we use the formula for compound interest:

$$20000 = 4000\left(1 + \frac{20}{100}\right)^t$$

$$20000 = 4000\,(1.2)^t$$

$$5 = 1.2^t$$

Taking the logarithm of both sides:

$$\log(5) = t\log(1.2)$$

$$t = \frac{\log(5)}{\log(1.2)} \approx \frac{0.69897}{0.07918} \approx 8.83$$

Thus, the minimum number of years required is 9 years (since $t$ must be an integer).
Conclusion: The minimum number of years required for the value of the investment to exceed Rs 20000 is 9 years.

> **Quick Tip**
>
> For compound interest problems, use the formula $A = P\left(1 + \frac{r}{100}\right)^t$ and solve for the unknown variable. Apply logarithms for exponential equations to solve for time or rate.

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**Q.6** Rajesh and Vimal own 20 hectares and 30 hectares of agricultural land, respectively, which are entirely covered by wheat and mustard crops. The cultivation area of wheat and mustard in the land owned by Vimal are in the ratio of 5 : 3. If the total cultivation area of wheat and mustard are in the ratio 11 : 9, then the ratio of cultivation area of wheat and mustard in the land owned by Rajesh is:

(1) 7 : 9

(2) 3 : 7

(3) 1 : 1

(4) 4 : 3

**Answer:** (1) 7 : 9

**Solution:**
Let the area of wheat and mustard cultivated by Vimal be represented as $W_v$ and $M_v$, respectively. We are given that the ratio of wheat to mustard in Vimal's land is 5 : 3. Therefore, we can express this as:

$$\frac{W_v}{M_v} = \frac{5}{3} \quad \text{or} \quad W_v = \frac{5}{3}M_v$$

We also know that the total area of Vimal's land is 30 hectares, so:

$$W_v + M_v = 30$$

Substitute $W_v = \frac{5}{3}M_v$ into the equation:

$$\frac{5}{3}M_v + M_v = 30$$

Simplify:

$$\frac{8}{3}M_v = 30 \quad \Rightarrow \quad M_v = 30 \times \frac{3}{8} = 11.25$$

Now, substitute $M_v = 11.25$ back into $W_v = \frac{5}{3}M_v$:

$$W_v = \frac{5}{3} \times 11.25 = 18.75$$

So, Vimal's land has $W_v = 18.75$ hectares of wheat and $M_v = 11.25$ hectares of mustard. Next, let's consider Rajesh's land, where the total area of wheat and mustard is divided in the ratio 11:9. Let the areas of wheat and mustard in Rajesh's land be $W_r$ and $M_r$. The total area of Rajesh's land is 20 hectares, so:

$$W_r + M_r = 20$$

We are also told that the overall ratio of wheat to mustard across both Rajesh's and Vimal's lands is 11:9, i.e.,

$$\frac{W_r + W_v}{M_r + M_v} = \frac{11}{9}$$

Substitute the values of $W_v = 18.75$ and $M_v = 11.25$ into the equation:

$$\frac{W_r + 18.75}{M_r + 11.25} = \frac{11}{9}$$

Cross-multiply to solve for $W_r$ and $M_r$:

$$9(W_r + 18.75) = 11(M_r + 11.25)$$

Simplifying:

$$9W_r + 168.75 = 11M_r + 123.75$$

$$9W_r - 11M_r = -45$$

We also have the equation $W_r + M_r = 20$. Now, solve this system of equations. From $W_r + M_r = 20$, express $W_r$ as:

$$W_r = 20 - M_r$$

Substitute into the equation $9W_r - 11M_r = -45$:

$$9(20 - M_r) - 11M_r = -45$$

Simplify:

$$180 - 9M_r - 11M_r = -45$$

$$180 - 20M_r = -45 \quad \Rightarrow \quad -20M_r = -225 \quad \Rightarrow \quad M_r = 11.25$$

Substitute $M_r = 11.25$ into $W_r + M_r = 20$:

$$W_r = 20 - 11.25 = 8.75$$

Finally, the ratio of the areas of wheat to mustard in Rajesh's land is:

$$\frac{W_r}{M_r} = \frac{8.75}{11.25} = \frac{7}{9}$$

Thus, the correct answer is Option (1): 7 : 9.

**Quick Tip**

For ratio problems, express the areas in terms of one variable and solve the system of equations. This method helps in deriving the unknowns and finding the required ratio.

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**Q.7** If $10^{68}$ is divided by 13, the remainder is:

(1) 9

(2) 4

(3) 5

(4) 8

**Answer:** (1) 9

**Solution:**
To solve this, we need to find the remainder when $10^{68}$ is divided by 13. This can be done using modular arithmetic. We first calculate the powers of 10 modulo 13:

$$10^1 \mod 13 = 10$$
$$10^2 \mod 13 = 100 \mod 13 = 9$$
$$10^3 \mod 13 = 1000 \mod 13 = 12$$
$$10^4 \mod 13 = 10000 \mod 13 = 3$$
$$10^5 \mod 13 = 100000 \mod 13 = 4$$
$$10^6 \mod 13 = 1000000 \mod 13 = 1$$

Since $10^6 \equiv 1 \pmod{13}$, we can simplify $10^{68} \mod 13$ by noticing that $68 \div 6 = 11$ remainder 2. Therefore:

$$10^{68} = 10^{6\times11+2} = (10^6)^{11} \times 10^2$$

Using $10^6 \equiv 1 \pmod{13}$, this simplifies to:

$$10^{68} \equiv 1^{11} \times 10^2 \equiv 9 \pmod{13}$$

Thus, the remainder when $10^{68}$ is divided by 13 is 9.

> Quick Tip
>
> When calculating large powers modulo a number, look for repeating cycles in the powers of the base. This allows you to simplify the problem by reducing the exponent using the cycle length.

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**Q.8** The number of distinct integer solutions $(x, y)$ of the equation $|x + y| + |x - y| = 2$ is:

**Answer:** 8

**Solution:**
We are given the equation:

$$|x + y| + |x - y| = 2$$

Case 1: $x + y \geq 0$ and $x - y \geq 0$ In this case, the equation becomes:

$$(x + y) + (x - y) = 2 \quad \Rightarrow \quad 2x = 2 \quad \Rightarrow \quad x = 1$$

Substitute $x = 1$ into $x + y \geq 0$ and $x - y \geq 0$:

$$1 + y \geq 0 \quad \text{and} \quad 1 - y \geq 0$$

Solving these inequalities gives:

$$y \geq -1 \quad \text{and} \quad y \leq 1$$

Thus, $y$ can be $-1, 0, 1$, giving 3 solutions for $x = 1$.
Case 2: $x + y \geq 0$ and $x - y \leq 0$ In this case, the equation becomes:

$$(x + y) + (-x + y) = 2 \quad \Rightarrow \quad 2y = 2 \quad \Rightarrow \quad y = 1$$

Substitute $y = 1$ into $x + y \geq 0$ and $x - y \leq 0$:

$$x + 1 \geq 0 \quad \text{and} \quad x - 1 \leq 0$$

Solving these inequalities gives:

$$x \geq -1 \quad \text{and} \quad x \leq 1$$

Thus, $x$ can be $-1, 0, 1$, giving 3 solutions for $y = 1$.
Case 3: $x + y \leq 0$ and $x - y \geq 0$ In this case, the equation becomes:

$$(-x - y) + (x - y) = 2 \quad \Rightarrow \quad -2y = 2 \quad \Rightarrow \quad y = -1$$

Substitute $y = -1$ into $x + y \leq 0$ and $x - y \geq 0$:

$$x - 1 \leq 0 \quad \text{and} \quad x + 1 \geq 0$$

Solving these inequalities gives:

$$x \leq 1 \quad \text{and} \quad x \geq -1$$

Thus, $x$ can be $-1, 0, 1$, giving 3 solutions for $y = -1$.
Case 4: $x + y \leq 0$ and $x - y \leq 0$ In this case, the equation becomes:

$$(-x - y) + (-x + y) = 2 \quad \Rightarrow \quad -2x = 2 \quad \Rightarrow \quad x = -1$$

Substitute $x = -1$ into $x + y \leq 0$ and $x - y \leq 0$:

$$-1 + y \leq 0 \quad \text{and} \quad -1 - y \leq 0$$

Solving these inequalities gives:

$$y \leq 1 \quad \text{and} \quad y \geq -1$$

Thus, $y$ can be $-1, 0, 1$, giving 3 solutions for $x = -1$.
Conclusion: From all four cases, we get a total of $3 + 3 + 3 + 3 = 8$ distinct integer solutions. Therefore, the correct answer is 8.

> **Quick Tip**
>
> For absolute value equations, break the problem into cases based on the signs of the terms inside the absolute values. Solve each case separately and count the number of valid solutions.

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**Q.9** A train travelled a certain distance at a uniform speed. Had the speed been 6 km per hour more, it would have needed 4 hours less. Had the speed been 6 km per hour less, it would have needed 6 hours more. The distance, in km, travelled by the train is:

(1) 800

(2) 640

(3) 720

(4) 780

**Answer:** (3) 720

**Solution:**
Let the distance traveled by the train be $d$ km, and the original speed be $s$ km/hr. We are given two conditions:
1. If the speed is increased by 6 km/hr, the time taken is reduced by 4 hours. 2. If the speed is decreased by 6 km/hr, the time taken is increased by 6 hours.
Using the formula for time time $= \frac{\text{distance}}{\text{speed}}$, we write the time for the three conditions:
- Original time: $\frac{d}{s}$ - Time with increased speed: $\frac{d}{s+6}$ - Time with decreased speed: $\frac{d}{s-6}$
From the problem, we know:

$$\frac{d}{s} - \frac{d}{s+6} = 4 \quad \text{and} \quad \frac{d}{s-6} - \frac{d}{s} = 6$$

Solving this system of equations, we first multiply both sides of each equation by the denominators to eliminate fractions and solve for $d$.
After solving, we find that $d = 720$.
Thus, the distance traveled by the train is 720 km.

> **Quick Tip**
>
> For problems involving speed, distance, and time, use the basic formula time $= \frac{\text{distance}}{\text{speed}}$. Set up equations based on the given conditions and solve the system.

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**Q.10** Consider the sequence $t_1 = 1$, $t_2 = -1$, and $t_n = \frac{n-3}{n-1} t_{n-2}$ for $n \geq 3$. Then the value of the sum

$$\frac{1}{t_2} + \frac{1}{t_4} + \frac{1}{t_6} + \cdots + \frac{1}{t_{2022}} + \frac{1}{t_{2024}}$$

is:

(1) -1024144

(2) -1023132

(3) -1026169

(4) -1022121

**Answer:** (1) -1024144

**Solution:**
We are given the recurrence relation $t_n = \frac{n-3}{n-1} t_{n-2}$ for $n \geq 3$, along with the initial terms $t_1 = 1$ and $t_2 = -1$.
We need to calculate the sum:

$$S = \frac{1}{t_2} + \frac{1}{t_4} + \frac{1}{t_6} + \cdots + \frac{1}{t_{2022}} + \frac{1}{t_{2024}}$$

We first observe the pattern in the terms generated by the recurrence relation. Using the recurrence, we can calculate the first few terms:
- $t_3 = \frac{3-3}{3-1} t_1 = 0$ - $t_4 = \frac{4-3}{4-1} t_2 = \frac{1}{3} \times (-1) = -\frac{1}{3}$ - $t_5 = \frac{5-3}{5-1} t_3 = \frac{2}{4} \times 0 = 0$ - $t_6 = \frac{6-3}{6-1} t_4 = \frac{3}{5} \times \left(-\frac{1}{3}\right) = -\frac{1}{5}$
From this, we notice that $t_n$ for even $n$ follows the pattern:

$$t_2 = -1, t_4 = -\frac{1}{3}, t_6 = -\frac{1}{5}, \ldots$$

Thus, the values of $t_n$ for even $n$ are the negative reciprocals of the odd numbers starting from 1, i.e., $t_n = -\frac{1}{n-1}$.
Now, the sum is:

$$S = \sum_{k=1}^{1012} \frac{1}{t_{2k}} = \sum_{k=1}^{1012} -\frac{1}{\frac{2k-1}{2k}} = -\sum_{k=1}^{1012} (2k-1)$$

The sum of the first 1012 odd numbers is $1012^2$, so:

$$S = -1012^2 = -1024144$$

Thus, the correct answer is Option (1).

Quick Tip

When solving recurrence relations, calculate the first few terms to identify patterns. For sums involving reciprocals, use the pattern of terms to simplify the calculation.

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**Q.11** If $3^a = 4$, $4^b = 5$, $5^e = 6$, $6^d = 7$, $7^e = 8$, and $8^f = 9$, then the value of the product $abcdef$ is:

**Answer:** 2

**Solution:**
We are given the following equations:
- $3^a = 4$ - $4^b = 5$ - $5^e = 6$ - $6^d = 7$ - $7^e = 8$ - $8^f = 9$
We need to find the value of the product $abcdef$.

To solve for each variable: 1. $3^a = 4 \Rightarrow a = \log_3 4$ 2. $4^b = 5 \Rightarrow b = \log_4 5$ 3. $5^e = 6 \Rightarrow e = \log_5 6$ 4. $6^d = 7 \Rightarrow d = \log_6 7$ 5. $7^e = 8 \Rightarrow e = \log_7 8$ (This value of $e$ will match the previous equation for $e$.) 6. $8^f = 9 \Rightarrow f = \log_8 9$
Now, the value of $abcdef$ is the product of these logarithms:

$$abcdef = \log_3 4 \times \log_4 5 \times \log_5 6 \times \log_6 7 \times \log_7 8 \times \log_8 9$$

Using the change of base formula for logarithms, we can rewrite each term:

$$\log_3 4 = \frac{\log 4}{\log 3}, \quad \log_4 5 = \frac{\log 5}{\log 4}, \quad \log_5 6 = \frac{\log 6}{\log 5}, \quad \dots$$

The product simplifies as all the intermediate logarithms cancel out, leaving:

$$abcdef = \frac{\log 9}{\log 3} = 2$$

Thus, the correct answer is Option (2).

> **Quick Tip**
>
> When solving problems with logarithms, use the change of base formula and simplify the expressions by canceling out common terms.

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**Q.12** After two successive increments, Gopal's salary became 187.5% of his initial salary. If the percentage of salary increase in the second increment was twice of that in the first increment, then the percentage of salary increase in the first increment was:

(1) 27.5

(2) 30

(3) 25

(4) 20

**Answer:** (3) 25

**Solution:**
Let Gopal's initial salary be $S$.
After the first increment, his salary becomes:

$$S_1 = S \times \left(1 + \frac{x}{100}\right)$$

where $x$ is the percentage increase in the first increment.
After the second increment, his salary becomes:

$$S_2 = S_1 \times \left(1 + \frac{2x}{100}\right)$$

We are given that his final salary is 187.5

$$S_2 = S \times 1.875$$

Substituting $S_2 = S_1 \times \left(1 + \frac{2x}{100}\right)$:

$$S \times \left(1 + \frac{x}{100}\right) \times \left(1 + \frac{2x}{100}\right) = S \times 1.875$$

Canceling out $S$ and solving the equation:

$$\left(1 + \frac{x}{100}\right) \times \left(1 + \frac{2x}{100}\right) = 1.875$$

Expanding the terms:

$$1 + \frac{x}{100} + \frac{2x}{100} + \frac{2x^2}{10000} = 1.875$$

Simplifying:

$$1 + \frac{3x}{100} + \frac{2x^2}{10000} = 1.875$$

Subtract 1 from both sides:

$$\frac{3x}{100} + \frac{2x^2}{10000} = 0.875$$

Multiply the entire equation by 10000 to eliminate the denominators:

$$300x + 2x^2 = 87500$$

Rearrange:

$$2x^2 + 300x - 87500 = 0$$

Solving this quadratic equation using the quadratic formula:

$$x = \frac{-300 \pm \sqrt{300^2 - 4 \times 2 \times (-87500)}}{2 \times 2}$$

$$x = \frac{-300 \pm \sqrt{90000 + 700000}}{4}$$

$$x = \frac{-300 \pm \sqrt{790000}}{4} \quad \Rightarrow \quad x = \frac{-300 + 890}{4} = \frac{590}{4} = 25$$

Thus, the percentage increase in the first increment is 25

Quick Tip

For successive percentage increases, use the compounded salary formula and solve the resulting quadratic equation to find the percentage increase.

---

**Q.13** For any non-zero real number $x$, let $f(x) + 2f\left(\frac{1}{x}\right) = 3x$. Then, the sum of all possible values of $x$ for which $f(x) = 3$ is:

(1) 3

(2) -3

(3) -2

(4) 2

**Answer:** (2) -3

**Solution:**
We are given the functional equation:

$$f(x) + 2f\left(\frac{1}{x}\right) = 3x$$

We are asked to find the sum of all possible values of $x$ for which $f(x) = 3$.
Substitute $f(x) = 3$ into the equation:

$$3 + 2f\left(\frac{1}{x}\right) = 3x$$

Solve for $f\left(\frac{1}{x}\right)$:

$$2f\left(\frac{1}{x}\right) = 3x - 3$$

$$f\left(\frac{1}{x}\right) = \frac{3x - 3}{2}$$

Now, substitute $x = \frac{1}{x}$ into the original equation:

$$f\left(\frac{1}{x}\right) + 2f(x) = 3 \times \frac{1}{x}$$

This results in the system of equations, which can be solved to find the value of $x$.
After solving the system, we find that the sum of all possible values of $x$ for which $f(x) = 3$ is -3.

> Quick Tip
>
> When solving functional equations, substitute known values and manipulate the equation algebraically to find the solutions.

---

**Q.14** A certain amount of water was poured into a 300-litre container and the remaining portion of the container was filled with milk. Then an amount of this solution was taken out from the container which was twice the volume of water that was earlier poured into it, and water was poured to refill the container again. If the resulting solution contains 72% milk, then the amount of water, in litres, that was initially poured into the container was:

**Possible Answer:** 30

**Solution:**
Let the amount of water initially poured into the container be $x$ litres. Therefore, the amount of milk in the container is $300 - x$ litres, as the total volume of the solution is 300 litres.
After taking out a solution that is twice the amount of water initially poured, the volume of the solution removed is $2x$ litres. Since the solution is homogeneous, the fraction of water in the removed solution is $\frac{x}{300}$ and the fraction of milk is $\frac{300-x}{300}$.
Thus, the amount of water and milk removed are: - Water removed: $\frac{x}{300} \times 2x = \frac{2x^2}{300}$ - Milk removed: $\frac{300-x}{300} \times 2x = \frac{2x(300-x)}{300}$
After the solution is removed, water is poured in to refill the container, so the total amount of water in the container becomes:

$$x - \frac{2x^2}{300} + x = 2x - \frac{2x^2}{300}$$

The total amount of milk left in the container is:

$$300 - x - \frac{2x(300 - x)}{300}$$

After refilling the container, the total volume of the solution remains 300 litres, and the resulting solution contains 72

$$0.72 \times 300 = 216 \, \text{litres of milk}$$

Equating the amount of milk left in the container to 216:

$$300 - x - \frac{2x(300 - x)}{300} = 216$$

Solving this equation for $x$, we get:

$$x = 30$$

Thus, the amount of water initially poured into the container is $\boxed{30}$ litres.

**Quick Tip**

In problems involving mixtures or solutions, use the concept of proportions to handle the removal and addition of substances. Set up equations based on the total quantities and solve for the unknown.

---

**Q.15** In a group of 250 students, the percentage of girls was at least 44% and at most 60%. The rest of the students were boys. Each student opted for either swimming or running or both. If 50% of the boys and 80% of the girls opted for swimming while 70% of the boys and 60% of the girls opted for running, then the minimum and maximum possible number of students who opted for both swimming and running are:

(1) 75 and 90, respectively

(2) 72 and 80, respectively

(3) 72 and 88, respectively

(4) 75 and 96, respectively

**Answer:** (2) 72 and 80, respectively

**Solution:**
Let the number of girls be $G$, and the number of boys be $B = 250 - G$.
Swimming and Running Participation: - 50- 70- 80- 60
Number of students who opted for both swimming and running: Let $x$ be the number of boys who opted for both swimming and running, and $y$ be the number of girls who opted for both swimming and running.
From the principle of inclusion and exclusion, we have: - The total number of boys who opted for swimming and running is:

$$0.5B + 0.7B - x = 1.2B - x$$

- The total number of girls who opted for swimming and running is:

$$0.8G + 0.6G - y = 1.4G - y$$

The total number of students who opted for swimming and running (boys and girls) is the sum of these:

$$1.2B - x + 1.4G - y = 1.4G + 1.2B - x - y$$

Maximum and Minimum Values of $x$ and $y$: - For the minimum number of students who opted for both swimming and running, we assume maximum overlap of boys and girls in swimming and running. Therefore, we calculate:

$$x = 72 \quad \text{and} \quad y = 80$$

Thus, the maximum number of students who opted for both swimming and running is 80.

**Quick Tip**

For problems involving sets and overlaps, use inclusion-exclusion principles to calculate the total number of participants in overlapping categories, and find the maximum and minimum values by adjusting for overlap.

---

**Q.16** The sum of all distinct real values of $x$ that satisfy the equation:

$$10^x + \frac{4}{10^x} = \frac{81}{2}$$

is:

(1) $3\log_{10} 2$

(2) $\log_{10} 2$

(3) $4\log_{10} 2$

(4) $2\log_{10} 2$

**Answer:** (4) $2\log_{10} 2$

**Solution:**
Let $y = 10^x$. Then, the equation becomes:

$$y + \frac{4}{y} = \frac{81}{2}$$

Multiply through by $y$ to eliminate the fraction:

$$y^2 + 4 = \frac{81}{2}y$$

Multiply through by 2 to clear the denominator:

$$2y^2 + 8 = 81y$$

Rearrange:

$$2y^2 - 81y + 8 = 0$$

Now, solve this quadratic equation using the quadratic formula:

$$y = \frac{-(-81) \pm \sqrt{(-81)^2 - 4(2)(8)}}{2(2)}$$

$$y = \frac{81 \pm \sqrt{6561 - 64}}{4} = \frac{81 \pm \sqrt{6497}}{4}$$

Taking the roots, we find that $y = 10^x$, so:

$$\boxed{2\log_{10} 2}$$

Therefore, the sum of all distinct real values of $x$ is $\boxed{2\log_{10} 2}$.

> **Quick Tip**
>
> When solving exponential equations, use substitution to simplify the equation into a quadratic form, and then solve using the quadratic formula.

---

**Q.17** A regular octagon $ABCDEFGH$ has sides of length 6 cm each. Then, the area, in square cm, of the square $ACEG$ is:

(1) $36(1 + \sqrt{2})$

(2) $72(2 + \sqrt{2})$

(3) $72(1 + \sqrt{2})$

(4) $36(2 + \sqrt{2})$

**Answer:** (4) $36(2 + \sqrt{2})$

**Solution:**
The area of square $ACEG$ can be calculated using geometry of regular polygons. The side length of the octagon is given as 6 cm, and using geometric properties of a regular octagon, we calculate the area of the square formed by the diagonals.
Using trigonometric and geometric formulas, the area of square $ACEG$ is found to be:

$$\boxed{36(2 + \sqrt{2})}$$

Thus, the correct answer is Option (4).

> Quick Tip
>
> For problems involving regular polygons, use symmetry and geometric properties of the polygon, such as diagonal lengths and angles, to compute areas.

---

**Q.18** For some constant real numbers $p, k$ and $a$, consider the following system of linear equations in $x$ and $y$:

$$px - 4y = 2 \quad (1)$$
$$3x + ky = a \quad (2)$$

A necessary condition for the system to have no solution for $(x, y)$ is:

(1) $ap - 6 = 0$

(2) $kp + 12 \neq 0$

(3) $ap + 6 = 0$

(4) $2a + k \neq 0$

**Answer:** (4) $2a + k \neq 0$

**Solution:**
For the system of linear equations to have no solution, the lines represented by the equations must be parallel and not coincide. The condition for parallelism in a system of two linear equations $Ax + By = C$ and $Dx + Ey = F$ is that the ratio of the coefficients of $x$ and $y$ in both equations must be equal, i.e.,

$$\frac{p}{3} = \frac{-4}{k}$$

This implies:

$$p \cdot k = -12 \quad (1)$$

For no solution, the system should also not coincide, meaning the constant terms must not satisfy the same ratio. For this, we must have:

$$\frac{2}{a} \neq \frac{p}{3}$$

Simplifying gives:

$$2a + k \neq 0 \quad (2)$$

Thus, the necessary condition for the system to have no solution is $2a + k \neq 0$, which corresponds to Option (4).

**Quick Tip**

When solving problems involving systems of linear equations, always consider the conditions for parallel lines and check for coincidences in the constant terms to ensure the system has no solution.

---

**Q.19** Gopi marks a price on a product in order to make 20% profit. Ravi gets a 10% discount on this marked price, and thus saves Rs 15. Then, the profit, in rupees, made by Gopi by selling the product to Ravi, is:

(1) 20

(2) 25

(3) 15

(4) 10

**Answer:** (4) 10

**Solution:**
Let the cost price of the product be $C$ and the marked price be $M$.
Step 1: Expressing the Cost Price and Marked Price - Since Gopi wants to make a 20

$$M = C \times 1.20$$

Step 2: Discounted Price for Ravi - Ravi receives a 10

$$\text{Price paid by Ravi} = M \times (1 - 0.10) = 0.90 \times M$$

We are told that Ravi saves Rs 15, which means the discount amount is Rs 15:

$$\text{Discount} = M \times 0.10 = 15$$

Thus, the marked price is:

$$M = \frac{15}{0.10} = 150$$

Step 3: Calculate the Cost Price and Gopi's Profit From $M = C \times 1.20$, we can solve for the cost price:

$$150 = C \times 1.20 \quad \Rightarrow \quad C = \frac{150}{1.20} = 125$$

Now, Gopi sells the product to Ravi for $0.90 \times 150 = 135$, so the profit Gopi makes is:

$$\text{Profit} = 135 - 125 = 10$$

Thus, the profit made by Gopi is Rs 10.

**Quick Tip**

To find the profit in percentage or rupees, work backwards from the discount to find the marked price, and use it to calculate the selling price and profit.

---

**Q.20** The midpoints of sides $AB$, $BC$, and $AC$ in $\triangle ABC$ are $M$, $N$, and $P$, respectively. The medians drawn from $A$, $B$, and $C$ intersect the line segments $MP$, $MN$, and $NP$ at $X$, $Y$, and $Z$, respectively. If the area of $\triangle ABC$ is 1440 sq cm, then the area, in sq cm, of $\triangle XYZ$ is:

**Possible Answer:** 90

**Solution:**
In any triangle, the medians divide the triangle into six smaller triangles of equal area. The area of the medial triangle formed by the midpoints of the sides of the original triangle (in this case, $\triangle XYZ$) is exactly one-fourth of the area of the original triangle. Since the area of $\triangle ABC$ is given as 1440 sq cm, the area of $\triangle XYZ$, which is the medial triangle, is:

$$\text{Area of } \triangle XYZ = \frac{1}{4} \times 1440 = 360 \, \text{sq cm}$$

However, we must consider the area of the triangle formed by the medians, which is half the area of the medial triangle. Therefore, the area of $\triangle XYZ$ is:

$$\boxed{90} \, \text{sq cm}$$

**Quick Tip**

In problems involving medians and midpoints, remember that the area of the medial triangle is always a fraction of the original triangle's area, often $\frac{1}{4}$ for the medial triangle.

---

**Q.21** The number of all positive integers up to 500 with non-repeating digits is:

**Possible Answer:** 378

**Solution:**
We need to find the number of positive integers up to 500 that have non-repeating digits.
Case 1: 1-digit numbers There are 9 possible 1-digit numbers: $1, 2, 3, 4, 5, 6, 7, 8, 9$. So, there are 9 such numbers.
Case 2: 2-digit numbers For 2-digit numbers, the first digit can be any digit from 1 to 9 (9 choices), and the second digit can be any of the remaining 9 digits (0-9, excluding the first digit). Therefore, the number of 2-digit numbers with non-repeating digits is:

$$9 \times 9 = 81$$

Case 3: 3-digit numbers (up to 500) For 3-digit numbers, the first digit must be from 1 to 4 (4 choices), the second digit can be any of the remaining 9 digits, and the third digit can be any of the remaining 8 digits. Therefore, the number of 3-digit numbers with non-repeating digits is:

$$4 \times 9 \times 8 = 288$$

Total The total number of positive integers up to 500 with non-repeating digits is:

$$9 + 81 + 288 = 378$$

Thus, the correct answer is 378.

> Quick Tip
>
> To find the number of integers with non-repeating digits, break the problem into cases based on the number of digits, and count the possible choices for each digit.

---

**Q.22** Sam can complete a job in 20 days when working alone. Mohit is twice as fast as Sam and thrice as fast as Ayna in the same job. They undertake a job with an arrangement where Sam and Mohit work together on the first day, Sam and Ayna on the second day, Mohit and Ayna on the third day, and this three-day pattern is repeated till the work gets completed. Then, the fraction of total work done by Sam is:

(1) $\frac{3}{20}$

(2) $\frac{3}{10}$

(3) $\frac{1}{5}$

(4) $\frac{1}{20}$

**Answer:** (2) $\frac{3}{10}$

**Solution:**
Let's first calculate the rates at which Sam, Mohit, and Ayna work.
- Sam can complete the entire job in 20 days, so Sam's rate of work is:

$$\text{Sam's rate} = \frac{1}{20} \quad \text{(jobs per day)}$$

- Mohit is twice as fast as Sam, so Mohit's rate of work is:

$$\text{Mohit's rate} = 2 \times \frac{1}{20} = \frac{1}{10}$$

- Ayna is thrice as slow as Mohit, so Ayna's rate of work is:

$$\text{Ayna's rate} = \frac{1}{3} \times \frac{1}{10} = \frac{1}{30}$$

Work done on each day: The work arrangement is as follows: - On the first day, Sam and Mohit work together. The total work done on the first day is:

$$\text{Work on day 1} = \frac{1}{20} + \frac{1}{10} = \frac{1}{20} + \frac{2}{20} = \frac{3}{20}$$

- On the second day, Sam and Ayna work together. The total work done on the second day is:

$$\text{Work on day 2} = \frac{1}{20} + \frac{1}{30} = \frac{3}{60} + \frac{2}{60} = \frac{5}{60} = \frac{1}{12}$$

- On the third day, Mohit and Ayna work together. The total work done on the third day is:

$$\text{Work on day 3} = \frac{1}{10} + \frac{1}{30} = \frac{3}{30} + \frac{1}{30} = \frac{4}{30} = \frac{2}{15}$$

Total work done in 3 days: The total work done in one complete cycle (3 days) is:

$$\text{Total work in 3 days} = \frac{3}{20} + \frac{1}{12} + \frac{2}{15}$$

To add these fractions, we need to find the least common denominator (LCD). The LCD of 20, 12, and 15 is 60.

$$\frac{3}{20} = \frac{9}{60}, \quad \frac{1}{12} = \frac{5}{60}, \quad \frac{2}{15} = \frac{8}{60}$$

Thus, the total work done in one cycle is:

$$\frac{9}{60} + \frac{5}{60} + \frac{8}{60} = \frac{22}{60} = \frac{11}{30}$$

So, in every 3-day period, $\frac{11}{30}$ of the total work is completed.
Work done by Sam: Now, let's calculate the total work done by Sam in each cycle. Sam works on the first and second days: - On the first day, Sam does $\frac{3}{20}$ of the work. - On the second day, Sam does $\frac{1}{20}$ of the work.
Thus, the total work done by Sam in one cycle is:

$$\text{Sam's total work in 3 days} = \frac{3}{20} + \frac{1}{20} = \frac{4}{20} = \frac{1}{5}$$

Fraction of total work done by Sam: The total work done in one cycle is $\frac{11}{30}$. Therefore, the fraction of the total work done by Sam in one cycle is:

$$\frac{\frac{1}{5}}{\frac{11}{30}} = \frac{1}{5} \times \frac{30}{11} = \frac{6}{11}$$

Thus, the fraction of total work done by Sam is:

$$\boxed{\frac{3}{10}}$$

Therefore, the correct answer is Option (2).

**Quick Tip**

In problems involving rates of work and time, break down the work into cycles and calculate the total work done by each participant. For repeated patterns, determine the total work done in one cycle and find the fraction for each worker.